The Unobserved Components Model

Abstract:

The Unobserved Components Model (UCM) is a popular forecasting technique which, like exponential smoothing and ARIMA, is local in nature, meaning that the more distant an observation is from the point of forecast, the less weight the distant observation carries in determining forecasts of the time series in question.

1. Working Mechanism:

The key feature of UCM is the decomposition of a time series into trend, seasonal, cycle and irregular components and the process used for the decomposition. Each component is formulated as a stochastically evolving process over time. This decomposition provides a better understanding of the dynamic characteristics of the series and the way these characteristics change over time.

The trend component typically represents the longer term developments of the time series of interest and is often specified as a smooth function of time. The recurring patterns within the years are captured by the seasonal component. The cyclic component can represent the dynamic features associated with the business cycle.

The UCM time series model has a natural state space representation.

2. State Space Models:

The concept of “state” is one of the fundamentals of the System Theory. Systems theory is the interdisciplinary study of systems. A system is an entity with interrelated and interdependent parts; it is defined by its boundaries and it is more than the sum of its parts (subsystem). Change in one part of the system affects other parts and the whole system, with predictable patterns of behavior.

The state of a system has as interpretation as the summary of the past observations of the system. The state taken together with the future system inputs determines all future states and system values. Now because of this feature of state space representation, the statistical treatment for the UCM model is based on the Kalman Filter and its related methods.

The UCM model generally breaks the entire time series dataset up as follows:

= +++ , ∼ NID(0, ), t = 1, . . . , n,

Where =observation at time t, =trend component at time t, = seasonal component at time t,= cyclical component at time t and =error component at time t.

We now go into the decomposition technique of each of the components.

3. Evaluating the Trend component:

There are two ways of evaluating the trend component:

a) Random Walk Model for Trend:

When the trend component is modeled as

, ∼ NIID (0, ), t = 1,..n, (NIID= normally independently identically distributed)

the trend is said to be the Random Walk (RW) trend model in the UCM. This model is especially appropriate for time series data that are flat and slow-turning.

b) Locally Linear Trend (LLT) Model:

When the trend is characterized by the following level and slope equations

, ∼ NIID (0, ), the Level Equation

= + , ∼NIID (0, ), the Slope Equation

Here represents the stochastic level of the trend while represents the stochastic slope of the trend.

4. Evaluating the Seasonal Component:

The seasonal component is generally evaluated using either of the two methods:

a) Stochastic Dummy Variable Seasonal Model:

If the seasonality in the data is such that its pattern sees a change over time, then this is the model which is used for estimating the seasonality. Let us consider a year and the seasonality of each month be assigned by,….. Now the model which is used in this scenario is:

, where ω ∼NIID (0, )

In this model the sum of the seasonal effects has a zero mean although their stochastic nature allows them to evolve either slowly over time (when is small) or quickly over time (when is large).

b) Deterministic Dummy Variable Seasonal Model:

If the seasonality in the data has the same pattern over time (i.e. =0), then we get the deterministic model

=0 for a yearly data.

5. Modeling the Cyclic Component:

The cyclic component of a time series dataset is usually calculated through either of the 2 processes:

a) Deterministic trigonometric cycle:

If the cyclic component is consistent with frequency λ , 0 < λ < π and does not change over time, then the cyclic component is written as

=α cos (λt) + β sin(λt) .

Here the parameters which have to estimated are α, β and λ.

Unfortunately, the cycles in economic and business time series data are scarcely ever as systematic as would be depicted in any one deterministic periodic function. So a single deterministic model can hardly ever capture the true picture of what actually is going on in reality. From Fourier analysis we know that fairly complex cyclical data can be written as a sum of a finite number of sinusoidal equations. Hence even the deterministic cyclic component equation may be of use in certain cases.

b) Stochastic trigonometric cycle:

A stochastic model for the cyclicity part is built by adding an error part to the deterministic model, thereby building the model

=α cos (λt) + β sin(λt)+, where ∼NIID (0, ).

From the Fourier analysis we know that a fairly complex cyclical data can be written as a sum of a finite number of sinusoidal equations. So to build an even refined model, the cyclic component can be shown as a combination of

=ρ.cos (λ) + ρ. (-sin (λ)).+

=ρ.(sin(λ)).+ρ.cos(λ).+

where 0 ≤ ρ ≤1 is a damping factor and the disturbances and ∼N (0, ) . This model can capture quite complex cyclical patterns in economic and business time series without introducing an abundance of parameters.

Once the models for forecasting are zeroed upon, the Kalman Filter is used for the purpose of estimating the parameters.

6. Forecasting Using UCM in R:

The “rucm” package is required for forecasting using ucm in R.

The function which is used for the forecast is “ucm”. This function has again the following parameters:

s.window, t.window, slope.

a) slope: Assigned True or False, depending on whether we want our model to have a slope or not.

b) s.window: A metric to control the smoothness of the seasonality. Higher the number, smoother the trend. Having a higher S. Window value is recommended.

c) t. window: A metric to control the smoothness of the trend.